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Differential Equation Models Derived from an Individual-Based Model Can Help to Understand Emergent Effects

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Abstract

We study a model of primacy effect on individual's attitude. Typically, when receiving a strong negative feature first, the individual keeps a negative attitude whatever the number of moderate positive features it receives afterwards. We consider a population of individuals, which receive the features from a media, and communicate with each other. We observe that interactions favour the primacy effect, compared with a population of isolated individuals. We derive a differential equation system ruling the evolution of probabilities that individuals retain different sets of features. The study of this aggregated model of the IBM shows that interaction can increase or decrease the number of individuals exhibiting a primacy effect. We verify on the IBM that the interactions can decrease the primacy effect in the conditions suggested by the study of the aggregated model. We finally discuss the interest of such a double-modelling approach (using a model of the individual based model) for this application.

Keywords:
Primacy Effect, Information Filtering, Agent-Based Model, Aggregated Model, Collective Effects of Interactions, Double-Modelling

Introduction

1.1

This paper focuses on a recently proposed simple individual based model (IBM) of the primacy effect (Deffuant and Huet 2007). The primacy effect (Asch 1946; Miller and Campbell 1959) occurs when somebody, who encounters a positive and then a negative message forms judgments which tend to be positive (of course positive and negative can be inverted). The model makes the assumption, that the primacy effect is related to the tendency to maintain internal consistency: the features contradicting the current global attitude tend to be filtered. Deffuant and Huet (2007) used individual based simulations to show that interactions can increase the primacy effect in a population.

1.2

This paper mainly focuses on the methodology used to study the IBM: the derivation of a differential equation model ruling over time the probability of individuals to belong to different groups. This approach can be called "double-modelling", because it needs to develop a model (differential equations) of an IBM (Deffuant 2004). The expected interest of the differential equation model is to provide explanations of the collective effects observed in IBM simulations, through an aggregated view of the IBM behaviour. We illustrate and discuss this on the particular case of the primacy effect IBM.

1.3

This approach is expressed and investigated in different researches. In ecology, Grimm (1999) encourages researchers to compare IBMs with aggregated models. Deffuant (2004) formalises a "double-modelling" which precisely advocates for the development of aggregate models to "theorise" IBMs. Focusing on attitude dynamics study, this double-modelling approach has been applied to the bounded confidence model (Deffuant et al. 2001,Hegselman and Krause 2002) by Ben-Naim P., Krapivsky L. et al (2003), Lorenz (2007), and Deffuant and Weisbuch (2007). Their purpose was to develop an exhaustive knowledge of this model asymptotic behaviour. Deffuant and Weisbuch (2007), using the same approach, improve the understanding of the extremist effect for this bounded confidence model. Martin, Deffuant et al (2004) applied this method to the study of a bit vector version of the bounded confidence model. They show the bigger limitation of the double modelling approach: the state space can be too large to be tractable. Edwards, Huet et al (2003), Ewards (2004), Huet, Edwards et al (2007) applied this method to study a stochastic IBM of binary behaviour diffusion individual model. They particularly aim at understanding the interaction effect in a random Erdös network. We present here a new application of this approach, which gives a particularly clear insight on the effect of the interactions.

1.4

The model, on which we apply this approach, uses the concept of attitude, understood as "a psychological tendency that is expressed by evaluating a particular entity with some degree of favour or disfavour" (Eagly and Chaiken 1998). It is widely observed that attitudes exert selective effects at various stages of information processing (Eagly and Chaiken 1998): information may be filtered (ignored) by the individuals. In his theory of cognitive dissonance, Festinger (1957) proposes some mechanisms for this selection: people seek out information that supports their attitudes and avoid information that challenges them, in order to minimise their cognitive dissonance. Following this theory, even if they assimilate information which contradicts their global attitude, people are reluctant to talk about it, because they avoid expressing their dissonance. Such selection mechanisms can imply sensitivity to the order of information delivery. In the present work, we are interested in people who are motivated to form an attitude about a particular object. Thus, they seek out relevant information and are sensitive to the media diffusion. Regarding the more recent literature, the attitude strength perspective assumes, among others, that the more you have knowledge on a particular object, the better you resist to a counter attitudinal attack, particularly when the messages require a cognitive effort and when you are motivated to think (Visser, Bizer and Krosnick 2006). This means that one tends not to consider an argument against one's own current attitude. Haugtvedt and Wegener (1994) conclude in their seminal paper: " *when participants* (to the experiment) *were motivated to elaborate on the message content, primacy effects occurred*."

1.5

Huet and Deffuant (2006) and Deffuant and Huet (2006, 2007) proposed a simple individual based model (IBM) of the individual primacy effect, which abstracts from the cited researches in social psychology. We particularly focussed on the following question: do the interactions between agents modify the likelihood of individual primacy effect in the population? With the simple model we consider, the answer is clearly positive. In some cases, the number of agents showing primacy effect is significantly higher, and in other cases significantly lower when agents interact than when they are isolated.

1.6

In Deffuant and Huet (2006), we derive and solve numerically a differential equation model of the individual based model, in order to better understand this particular impact of the interactions exposed to a short message (comprising a major negative feature and two positive features). It stressed that interactions favour the broadcasting of the major feature, which increases its probability to be received, thus giving an advantage to globally negative attitudes. Moreover, it appears that the differential equation model cannot reproduce the individual based model results when the frequency of the diffusion of the message by the media is too weak.

1.7

This paper extends this study by considering a more complex message, including two major negative and three positive features. We show that in this case, interactions can also decrease the number of individuals exhibiting the primacy effect. We study again this particular configuration through the corresponding differential equation model.

1.8

First, we describe the individual-based model and the impact of the interactions. In section 3, we present the methodology to build the differential equation model of the IBM and apply it to the case of a population exposed to a neutral complex message, composed of two major negative and three minor positive features. The analysis of the differential equation model allows us to understand better the impact of interactions and to assume that in some cases, interactions tend to decrease the primacy effect. In Section 4, we derive a new differential equation model, to test this assumption, and check its compatibility with the IBM. Finally, we conclude and discuss the benefit from the double-modelling approach.

The individual based model (IBM)

The dynamics of attitudes

2.1

Our model is strongly inspired by the dissonance theory (Festinger 1957) on the one hand, and on Allport's (1947) work on rumor diffusion on the other hand. To summarise, we assume that individuals avoid incongruent information, and, keep only important information.

2.2

We consider a population of N individuals forming a global attitude about an object. We define this object by a set of features $F = (1,2,\ldots, d)$, which are associated with positive or negative real values ($u_1,\ldots,u_j,\ldots,u_d$) with $u_j \in \Re$. An individual can have a partial view of the object, in which case it has a real value for some features and nil for others. To simplify we use feature instead of feature value in the following.

2.3

The model is based on the congruency principle. A feature is congruent when it has the same sign as the individual's global attitude to the object, incongruent otherwise.

2.4

An individual *i* is characterised by:

- g: An initial attitude (suppose the same for all individuals in the following).
- L_i: A subset of F containing the features currently retained by the individual (empty at the beginning).
- $G_i = g + \sum_{j \in L_i} u_j$: The global attitude about the object (related to information integration theory of Anderson (1971)).
- Θ_i : A threshold defining the absolute value since when an incongruent feature is judged enough high not to be filtered.
- A neighbourhood corresponding to the subset of individuals with whom *i* can communicate.

The dynamics of the model have four main aspects:

1. **Exposure to feature values.** We suppose that, at each time step:
   - A media sends a randomly chosen feature to the individual following a delivery frequency *f*, which is, on average, the number of individuals who are exposed to a feature per iteration.
   - An individual is exposed to feature values proposed by its neighbours during regular meeting (see 2.2.2.1. for details).
2. **Selective retaining**: The dynamics of filtering are determined by the individual incongruence threshold Θ_i. Being told about feature *j*, the individual *i* will react as follows:
   - If *j* is congruent → *i* "retains" the feature *j*. This means that *j* is added to *Li* (if L_i does not include *j* yet),
   - If *j* is incongruent:
     - if $| u_j | > \Theta_i$ → *i* "retains" the feature *j*;
     - otherwise *i* "ignores" the feature *j*. This means that *j* is filtered (not added to L_i).
3. **Selective emission:** individuals only talk about congruent features
4. **Computation of attitude:** an individual computes its global attitude each time it retains a new feature. As presented in the characteristics of the individual, the global attitude to the object is the sum of the feature values the individual retained and its initial attitude, g.

2.5

In the following, we consider that the individual incongruence threshold Θ_i is a constant Θ which is the same for all individuals of the population. Deffuant and Huet (2007) have considered various choices for Θ_i, which can be dynamic. They showed the particular interaction effect we are interested in occurs for all of the various studied Θ_i.

Impact of interactions on the primacy bias

2.6

As explained in Deffuant and Huet (2007), by an analysis of the definition of the dynamics presented above, we have to distinguish two main cases: a first very simple one, which is not sensitive to order of feature exposure, and a second one, which is sensitive to this order. Thus, when the model is sensitive to delivery order, the interactions, modifying this order, can have a particular effect on the final state of the population.

Individual trajectories can be sensitive to the order of feature exposure

The trajectories with a message including major negative and minor positive features

2.7

We now consider an individual with an initial attitude $g > 0$, and an object with at least one negative feature of absolute value higher than Θ (called major incongruent feature), and positive features lower than Θ (called minor congruent feature). Notice that the same reasoning can be done with inverted signs. In this case, the final attitude depends on the reception order:

- If the individual receives the negative feature first, if g is low enough, it can change its global attitude, and the positive features become incongruent. As they are lower than Θ, they are not retained.
- If the individual receives the positive features first, they are necessarily retained.

When the individual attitude is sensitive to the feature reception order, we can observe primacy effect: first few received features define the individual's attitude sign.

2.8

This leads us to define a more concrete example, used in all the following. We suppose that the initial attitude g is positive. Then we consider an object described by 5 features: two major negative ones, valued at $-U$, such that $U > \Theta$, and three positive ones, valued u, such that $u < \theta$. we suppose that the object is globally neutral, that is: $3u - 2U = 0$. For instance, we choose $U = 6$ and $u = 4$, with $\Theta = 5$. We are interested in this paper in a more complex "message" example than in Deffuant and Huet (2007), which treats of a 3-feature message composed from one major negative feature and two minor positive features. One more time, remain that this more complex message of 5 features is the one used in the following to study the individual based model dynamics.

2.9

Figure 1 shows the evolution of a global attitude, for a particular reception order of the features. Initially, the individual has an attitude $g = 6.5$. First, it receives a positive feature, which is retained because it is congruent, and its attitude increases to 10.5. Second, it then receives a negative feature, which is incongruent, but it is retained because its absolute value is higher than the threshold, and its attitude decreases to 4.5. Next, it receives the second negative feature, which is incongruent and also retained and its attitude decreases to -1.5. It is then exposed to the fourth and the fifth positive features, which are incongruent with an absolute value below the threshold, and therefore they are not retained. Its attitude thus does not change anymore. It has finally a negative attitude although the object is globally neutral. On figure 2, the individual receives firstly the three positive features consecutively, and then the two major negative features. All features are retained in this case, and the attitude follows another trajectory, leading to a final positive attitude.

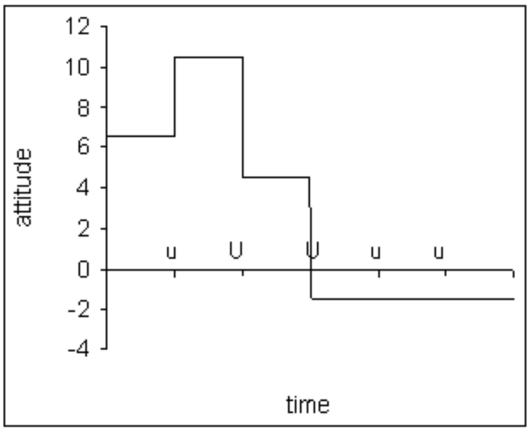

Figure 1. Examples of temporal evolution of the attitude of an individual with $g = 6.5$. On the left, the individual receives the features in order *u, U, U, u, u*. Its final attitude is negative

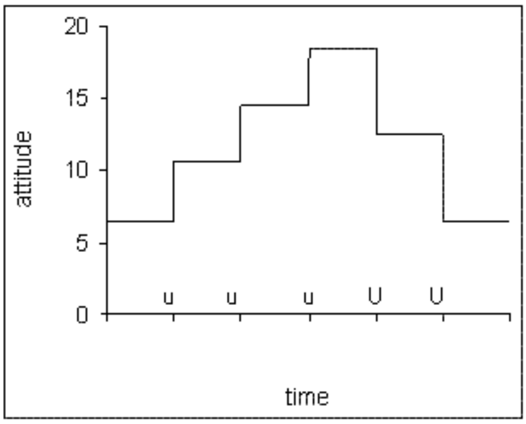

Figure 2. Trajectory for order of exposure *u, u, u, U, U*, and the final attitude is positive

2.10

Figures 3 shows the ten possible trajectories of attitude. The first, the two first, or at the most, the three first features determine the final sign of attitude: this is the primacy effect.

2.11

If we consider a population of isolated individuals, each individual trajectory has the same probability of occurring. It is thus very easy to predict the final part of negative versus positive people. It simply corresponds to the relative part of trajectories leading to a final negative attitude. From the figures 3, we can predict the final state of the population: 70 % of individual with a final positive attitude, because we observe 7 final positive trajectories out of 10 total trajectories (the presented seventeenth trajectories), and, 30 % of individual with a final negative attitude, because we observe 3 final negative trajectories out of 10 total trajectories (the last three trajectories of figures 3).

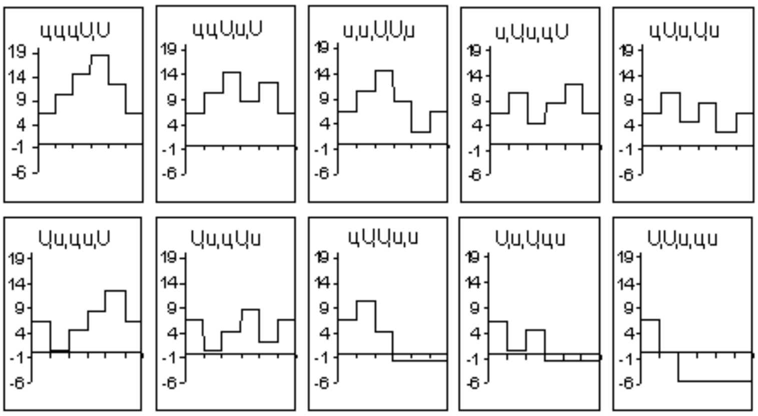

Figure 3. The 10 possible individual trajectories, for case g = 6.5, 5 features composed of 2 U and 3 u with U = −6 and u = +4. Three trajectories over ten lead to a final negative attitude

2.12

We know that the equiprobability of exposure is true for the media we chose. Nevertheless, when individuals interact, an individual can be exposed to a feature proposed by the media or by another individual. The interactions can modify the probability of presentation, and consequently the part of the population with a final negative attitude. We now investigate this impact of interactions.

The impact of interactions on final attitudes

2.13

Deffuant and Huet (2006, 2007), showed that interactions can increase the final part of the population with a negative attitude. This is the case for a message, composed of three features with one major negative attitude U and two minor positive attitudes u when g has a value in a range from 0 to u. We are now looking for the same "increase" effect of the interaction between people for our more complex neutral 5-feature message.

2.14

Before that, let us describe in more details the model of interactions.

Interaction model

2.15

The interaction mechanism is very simple. The aim is to ensure that, on average, one individual meets another individual in each iteration. As it is a stochastic process, it remains possible that one individual does not meet any other, or meets several others during an iteration.

2.16

We consider two cases:

- an individual talks only about the congruent features it retained (only congruent feature transmission);
- an individual talks about all the features it retained (any feature transmission).

2.17

The complete algorithm, containing exposure to the diffusion by media and exposure from interaction is:

For a population of *N* individuals, at each time step:
N times repeat:

- *Media diffusion*. choose individual *i* at random with probability *f*, choose feature *j* at random in the object, send feature *j* to individual *i*.
- *Interactions*: choose couple of individuals (*i, j*) at random:
  - *Only congruent feature transmission case*: *i* tells *j* about one of its randomly chosen congruent features
  - *All feature transmission case*: *i* tells *j* about one of its randomly chosen features (congruent or not).

Interactions can change the number of primacy effects

2.19

We run simulations of our IBM with our 5-feature message and an initial positive attitude $g = 2.5$. We have $0 < g < u$. Such an attitude value should allow us, following what we have found in Deffuant and Huet (2007) to observe that interactions increase the population part exhibiting the primacy effect. For these parameters, figure 4 shows a comparison between the number of final negative attitudes for isolated individuals and this number for interacting individuals (for both transmission of congruent features only, and transmission of all features). We observe that interactions induce more negative final attitudes than the isolated case. Indeed, we obtain 83% of negative individuals with transmission of only congruent features, but only 70% for isolated individuals. This impact of interactions is even higher when individuals can transmit any retained feature, even though it is not statistically significant.

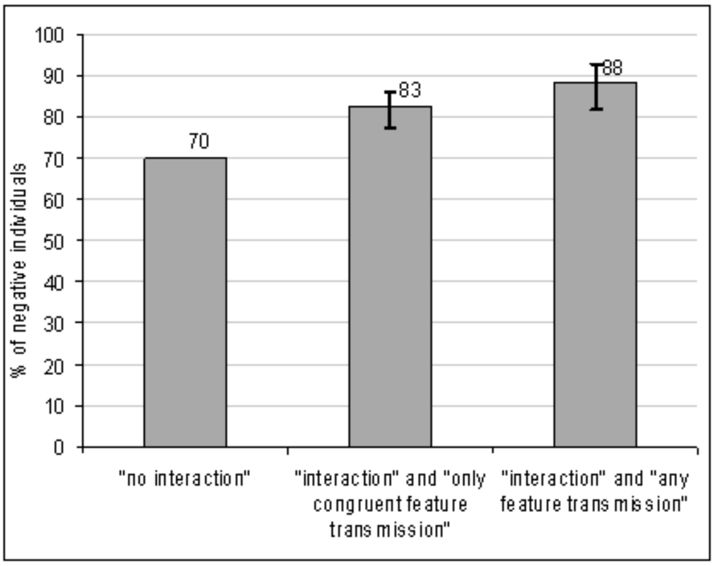

Figure 4. Final percentage of negative individuals, averaging on 100 replicas, for $g = 2.5$ for three various dynamics: isolated individuals; interacting individuals transmitting only congruent features and interacting individuals transmitting any feature

2.21

In Deffuant and Huet (2006) for the 3-feature message, we used a differential equation model of the IBM to analyse this impact of interactions. We are now considering again this approach in the case of the 5-feature message.

Bird's-eye view of the IBM for complex "interaction" cases

3.1 We now consider the probability of individuals to belong to different groups (defined by the features they retained) over time. We assume that the dynamics of these probabilities (corresponding to a infinite population) reflect the evolution of the IBM. We already know that this assumption has a limited validity: Deffuant and Huet (2006) showed that it supposes a significant level of media diffusion. For weak frequency of diffusion (0.001 and less), there are many time steps without any diffusion from the media, which contradicts the assumptions behind the differential equation model. Therefore, we now suppose that the diffusion frequency is high enough.

3.2 First, we write down the differential equations ruling the probabilities to belong to the different groups, solve it numerically and compare it with the IBM. Secondly, we analyse the differential equation model to learn more about the interaction impact. In the following, we also use "aggregated model" to denote the differential equation model.

Building the model

3.3 The general idea to build the aggregated model is to consider groups of agents and to define the transfer equations ruling the flows of probability densities between the groups. First, we need to determine the groups, and the possible flows between them.

3.4 A group is defined by a possible list of retained features, which may appear at any moment of the simulation. The list of feature retained depends on the order of feature exposure. Thus, we have to begin with the study of what the different exposure orders imply. Table 1 lists all the various orders possible for our message (defined in 2.2.1.1.). It also shows that the final global attitude sign of an individual exposed to a particular order of features depends on the value of g. Thus, considering all the ten possible orders, the initial attitude g splits into different value segments to define a unique distribution of final global attitude sign on all orders.

3.5 The table shows that the primacy effect can be observed (i.e. that individuals are sensitive to event order) for $0 \leq g < 3\ u$ because some trajectories are finally positive while others are negative. For this given particular message, the exhaustive study of the interaction effect in complex cases needs to build six different aggregated models of group dynamics. Indeed a value segment of g defines a particular final distribution of positive and negative global attitude and, thus, as we will see later, a particular given simplification of groups to model. Each segment defining a particular list of groups, each segment leads to a particular aggregated model.

Table 1: Final sign of the global attitude for the ten different trajectories and all different values of g

Exposure order	$g < 0$	$0 \leq g < 0.5u$	$0.5u \leq g < u$	$u \leq g < 1.5u$	$1.5u \leq g < 2u$	$2u \leq g < 3u$	$g \geq 3u$
UUuuu	–	–	–	–	–	–	+
UuUuu	–	–	–	–	–	+	+
UuuUu	–	–	–	–	+	+	+
UuuuU	–	–	–	–	+	+	+
uUUuu	–	–	–	–	–	+	+
uUuUu	–	–	–	+	+	+	+
uUuuU	–	–	+	+	+	+	+
uuUUu	–	–	–	+	+	+	+
uuUuU	–	+	+	+	+	+	+
uuuUU	–	+	+	+	+	+	+

3.6 We select one set of values of g to construct the corresponding aggregated model: $u \leq g < 1.5\ u$. Figure 5 shows the transition graph of the different groups to model. In this case, 7 groups have to be considered. We start from a group having no features, able to receive a U or a u. In one hand, receiving a U implies the individual will always has a final negative global attitude, whatever you receive after. Thus, the second group is the one of people having received U at first. In the other hand, receiving a u is a third group to consider. As this third group does not allow the decision about the final global attitude sign, we continue to develop the branch. Having received *u*, it is possible to receive *u* or *U* ...We continue until each branch can be stopped because it defines without ambiguity the final sign of the global attitude.

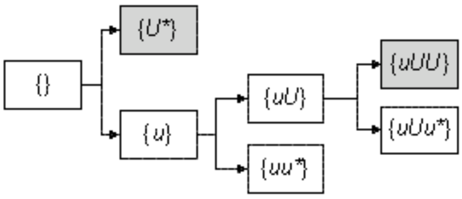

Figure 5. The graph of transitions between the groups for $u \le g < 1.5\ u$, defined by the set of retained features. The groups with a negative attitude are in grey

3.7

The second stage of the modelling approach is to determine the flow through each transition (i.e. each considered group). This requires evaluating the probability that the agents in each group retain a feature, which makes them change their group. This probability is directly related to the features, which are sent by each group. This is broken down in table 2 in the case of transmission of congruent features only. For example, you can read in the third column of the table 2 that all individuals who have received a *U* at first only talk to others about feature *U*.

Table 2: communicated features for each group in the case $u \le g < 1.5\ u$ or *U*

Group	Media	{*U**}	{*u*}	{*Uu*}	{*uu**}	{*uUU*}	{*uUu**}
Communicated features	*U*, *u*	*U*	*u*	*U*	*u*	*U*	*U*

3.8

This work done, it is possible to write down the differential equations for each group, summing up the flows in and subtracting the flows out the group. For $u \le g < 1.5\ u$, we get:

$$
\begin{aligned}
\frac{dS_0}{dt} &= -S_0\left(f + S_u + S_{U*} + S_{uU} + S_{uu*} + S_{uUU} + S_{uUu*}\right)\\
\frac{dS_{U*}}{dt} &= S_0\left(\frac{2f}{5} + S_{U*} + S_{uUU}\right)\\
\frac{dS_u}{dt} &= S_0\left(\frac{3f}{5} + S_u + S_{uu*} + S_{uUu*} + S_{uU}\right) - S_u\left(\frac{4f}{5} + S_{U*} + S_{uUU} + \frac{2}{3}\left(S_u + S_{uU} + S_{uUu*} + S_{uu*}\right)\right)\\
\frac{dS_{uU}}{dt} &= S_u\left(\frac{2f}{5} + S_{U*} + S_{uUU}\right) - S_{uU}\left(\frac{3f}{5} + \frac{1}{2}\left(S_{U*} + S_{uUU}\right) + \frac{2}{3}\left(S_u + S_{uUu*} + S_{uu*} + S_{uU}\right)\right)\\
\frac{dS_{uu*}}{dt} &= S_u\left(\frac{2f}{5} + \frac{2}{3}\left(S_u + S_{uUu*} + S_{uu*} + S_{uU}\right)\right)\\
\frac{dS_{uUU}}{dt} &= S_{uU}\left(\frac{f}{5} + \frac{1}{2}\left(S_{U*} + S_{uUU}\right)\right)\\
\frac{dS_{uUu*}}{dt} &= S_{uU}\left(\frac{2f}{5} + \frac{2}{3}\left(S_u + S_{uU} + S_{uu*} + S_{uUu*}\right)\right)
\end{aligned}
\qquad (1)
$$

with:
S_0: proportion of individuals with a void list of retained features,
S_u: proportion of individuals with a list of retained features containing only *u*,
S_{U*}: proportion of individuals following all trajectories beginning with *U*,
S_{uU}: proportion of individuals with a list of retained features containing only u at first and *U* at second
S_{uu*}: proportion of individuals following all trajectories beginning with *uu*.
S_{uUU}: proportion of individuals following all trajectories beginning with *uUU*.
S_{uUu*}: proportion of individuals following all trajectories beginning with *uUu*.
f: frequency of media feature communication.

We compute finally the evolution of groups at the end by calculating, for each group S_G with $G \in \{0, u, U^*, uU, uu^*, UU, uUu^*\}$:

$$S_G = S_G + \frac{dS_G}{dt}dt \qquad (3)$$

3.9

The systems can be simulated considering different values for *dt*. After having tested various possible values for *dt*, it appears *dt* = 0.1 is weak enough to approximate correctly the differential equation system. Indeed, the results for *dt* = 0.1 are exactly the same as the one obtained with smaller values of *dt*.

Comparison of the aggregated model with the individual-based model

3.10

For the IBM, we consider a population of 5041 individuals. From runs of the IBM and aggregated model, it results that the part of final negatives in the population for $u < g < 1.5 u$ is 53.3% on average for the IBM (with a minimum of 45% and a maximum of 64% on 100 replicas) and 53.2% for the aggregated model with *dt* = 0.1. It appears that the aggregated model gives an accurate approximation of the average number of negative individuals in the population.

3.11

Figure 6 shows the evolution of the part of each group during the simulation for the IBM on the one hand, and for the aggregated model on the other hand. Group results for the IBM are built by a concatenation of the individual level results. One more time, we observe that both models, IBM and aggregated, give very close results.

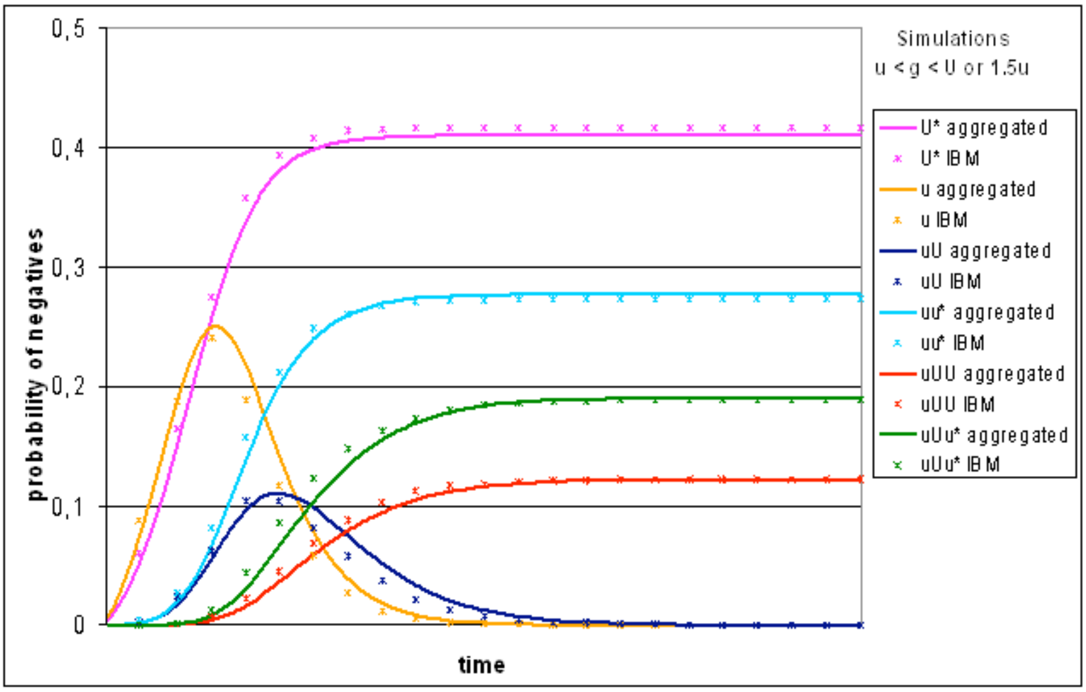

Figure 6. Comparison of trajectories of each groups of aggregated and IBM model. One measure of the IBM's replicas is put all the ten measures of the aggregated model

Using the aggregated model to better understand the individual-based model

3.12

From the results of the aggregated model, we obtain the proportion at each time step of the negative feature *U* communicated during interactions. This proportion of *U* emission by interaction can be compared with the proportion of *U* emission from the media. Figure 7 shows this comparison for the particular message and initial value of attitude we study.

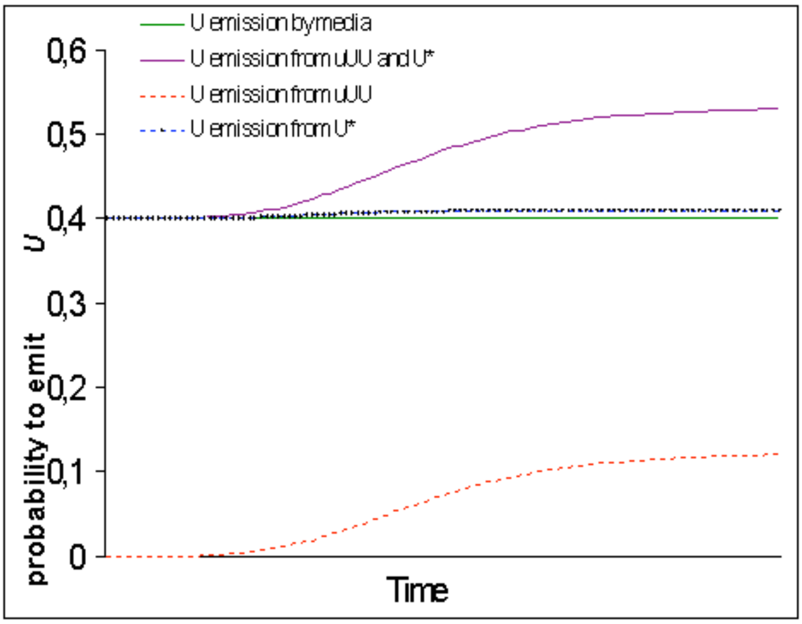

Figure 7. Comparison of probability of *U* emission due to interaction with the probability of *U* emission due to medium for $u \leq g < 1.5\ u$

3.13

We see on figure 7 that the global probability of *U* emission by interaction begins at a value equal to probability of *U* emission by medium. It increases with the *U* emission from the negative group *uUU*. Thus, referring to figure 5, we can remark that the presence of the negative group *uUU*, following positive groups in the tree whereas no positive groups follows a negative one, induces a diffusion advantage in favour of *U*. This explains the increase of negative final states.

3.14

Can such an advantage be in favour of *u* for a different value of *g*? The aggregated approach will now help us to answer to this question.

The aggregated model helping to predict the IBM

4.1

From the previous analysis, and from the observation of the table 1, we select the segment of initial attitude $2\ u \leq g < 3\ u$ for which we notice that the transition branch beginning with *U* leads to positive and negative groups whereas the one beginning with *u* only leads to positive groups.

The differential equation model for $2\ u \leq g < 3\ u$

4.2

Figure 8, showing the transition graph of the different groups to model, illustrates our attitude segment choice. Indeed, one can anticipate that group *Uu** will increase the frequency of positive feature communication.

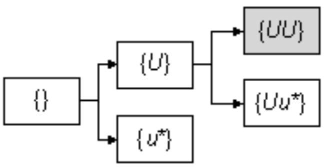

Figure 8. The graph of transitions between the groups for $2\ u \leq g < 3\ u$, defined by the set of retained features. The groups in grey have a negative attitude

4.3

Table 3 determines the flow through each transition.

Table 3: communicated features for each group in the case $2\ u \leq g < 3\ u$ or $2\ U$

Group	Media	{*U*}	{*u**}	{*UU*}	{*Uu**}
Communicated features	*U*, *u*	*none*	*u*	*U*	*u*

Then we can write down the differential equations for this new case, $2u \le g < 3u$:

$$\begin{aligned}
\frac{dS_0}{dt} &= -S_0(f + S_{u*} + S_{UU*} + S_{Uu*}) \\
\frac{dS_{u*}}{dt} &= S_0(\frac{3f}{5} + S_{u*} + S_{Uu*}) \\
\frac{dS_U}{dt} &= S_0(\frac{2f}{5} + S_{UU*}) - S_U(\frac{4f}{5} + \frac{S_{UU*}}{2} + S_{u*} + S_{Uu*}) \\
\frac{dS_{UU*}}{dt} &= S_U(\frac{f}{5} + \frac{S_{UU*}}{2}) \\
\frac{dS_{Uu*}}{dt} &= S_U(\frac{3f}{5} + S_{u*} + S_{Uu*})
\end{aligned} \tag{2}$$

with :
S_0: proportion of individuals with a void list of retained features,
S_U: proportion of individuals with a list of retained features containing only *U*,
*S u**: proportion of individuals following all trajectories beginning with *u*,
*S UU**: proportion of individuals following all trajectories beginning with *UU*,
*S Uu**: proportion of individuals following all trajectories beginning with *Uu*.
f: frequency of feature diffusion by the media.

Comparison of the aggregated model with the individual-based model

4.4

As previously, we consider a population of 5041 individuals for the IBM. From runs of the IBM and aggregated models, it results that the part of final negatives in the population for $2u \le g < 3u$ is 0.4% on average for the IBM (with a minimum of 0.2% and a maximum of 0.8% on 100 replications) and 0.4% for the aggregated model with $dt = 0.1$. The final part of negatives is very weak because, in most cases, individuals do not consider the negative features. Moreover, the interactions increase the diffusion of the positive features. Thus, it is very difficult for an individual to receive the negative information at an early stage, which is the only way for it to remain negative.

4.5

Figure 9 shows the evolution of the part of individuals in each group during the simulation for the IBM on the one hand, and for the aggregated model on the other hand. One more time, we observe that both models, IBM and aggregated, give very close results.

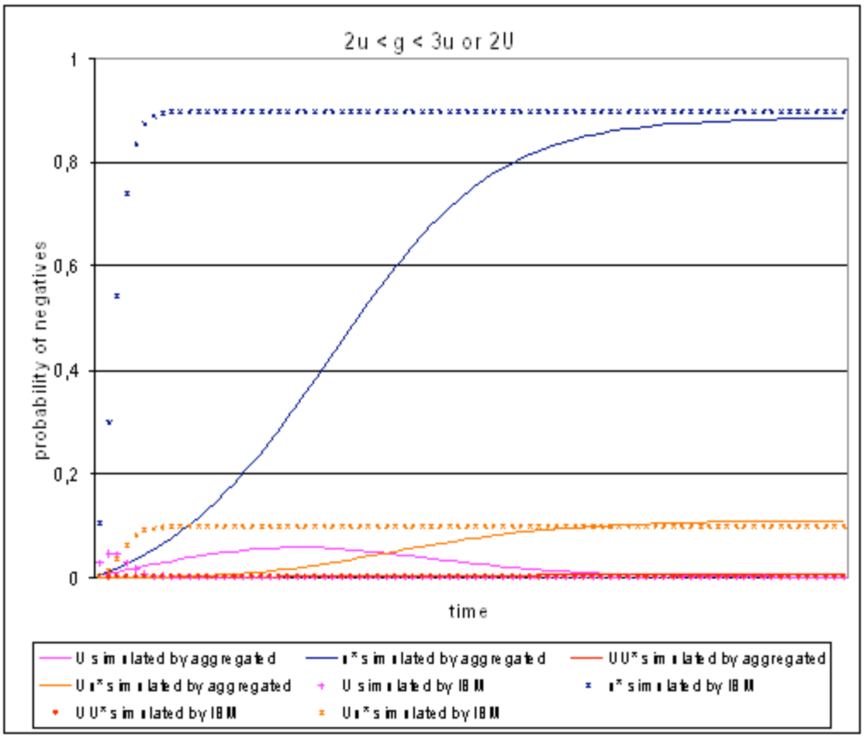

Figure 9. Comparison of trajectories of each groups of aggregated and IBM model for $2u \le g < 3u$. One measure of the IBM's replicas is put all the ten measures of the aggregated

4.6

As previously, from the results of the aggregated model, we obtain the proportion at each time step of the negative feature *U* communicated during interactions. This proportion of *U* emission by interaction can be compared with the proportion of *U* emission from the media. Figure 10 shows this comparison for 2 $u \leq g < 3$ u.

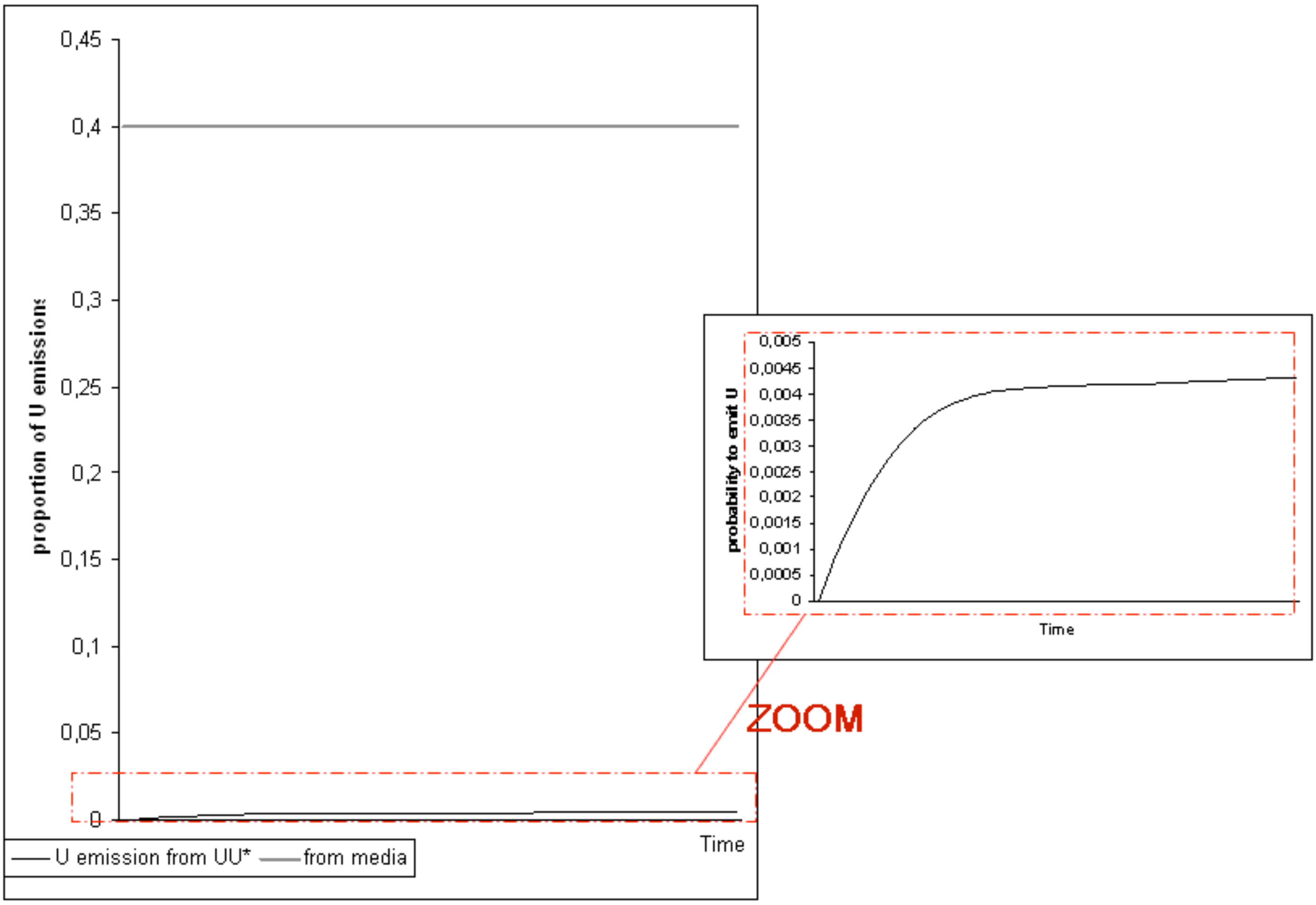

Figure 10. Comparison of probability of *U* emission due to interaction with the probability of *U* emission due to medium for the "decrease" interaction" effect case

We see on figure 10 that the probability of *U* emission by interaction (from 0 to 0.0042) is always lower than the probability of *U* emission by the media (0.4).

4.7

Now, we can generalize our conclusions:

- From the observation of table 1, we deduce that, for $0 \leq g < 1.5$ u, the interactions increase the primacy effect diffusion because the transition branch beginning with *u* contains negative groups whereas the transition branch beginning with *U* does not contain positive groups. Thus, for these three segments, the diffusion of the negative feature *U* is increased by interactions.
- Following the same reasoning, for 1.5 $u \leq g < 3$ u, the interactions decrease the diffusion of primacy effect. We studied above the case 2 $u \leq g < 3$ u. For 1.5 $u \leq g < 2$ u, things are less clear. However, we can observe, from table 1, that half of exposure orders beginning with the negative feature *U* quickly lead to a positive global attitude (for the second received feature), whereas only one on six exposure orders beginning with the positive feature *u* leads less quickly to a negative global attitude (from the third received feature). Thus, with the transmission of only congruent features, the diffusion of *u* is favoured.

4.8

A comparison between the IBM and the isolated case for all the considered initial values of *g* confirms our generalization. Figure 11 shows the number of final negative attitudes for the IBM with interactions (with transmission of only congruent features and transmission of all features) and in the isolated case. The isolated case is represented by dark bars, and the interaction cases (average results on 100 replicas) are represented in grey bars (transmission of all features) and in white bars (transmission of congruent features only).

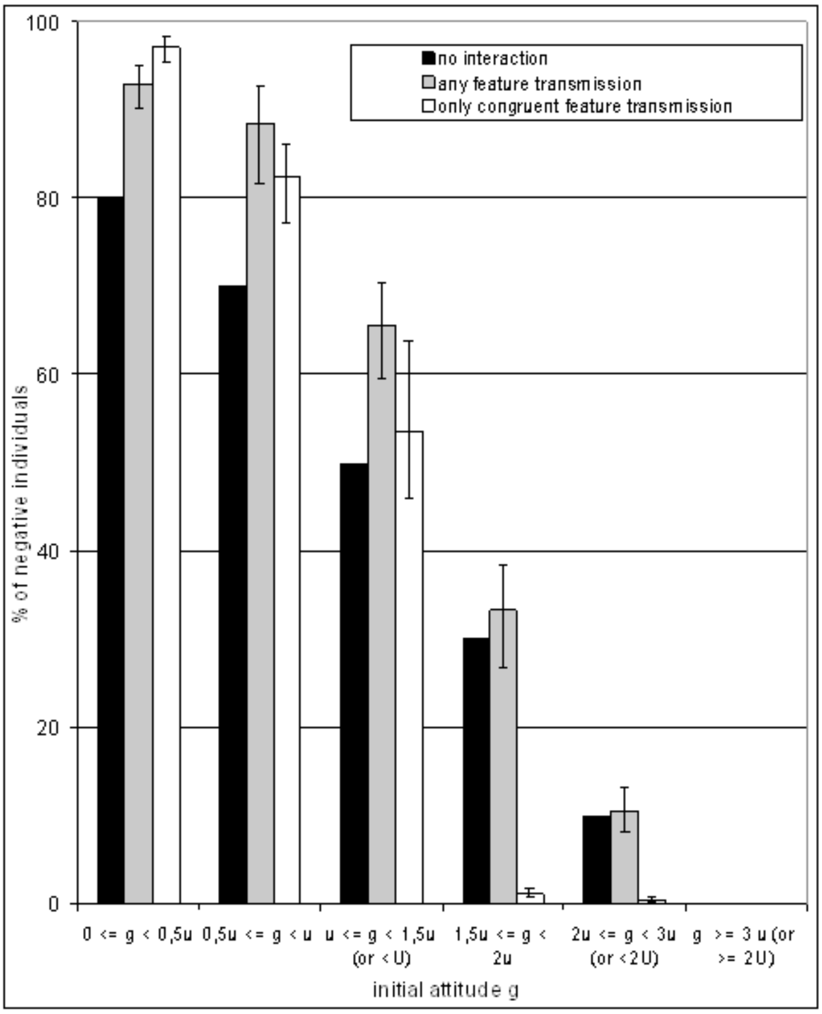

Figure 11. Final percentage of negative individuals for various values of *g* and for three dynamics: isolated individuals (no interaction), interaction with any feature transmission, interaction with only congruent transmission (average on 100 replicas). The errors bars represent the minimum and maximum values on 100 replicas

We observe on the left, for $0 \leq g < 1.5\ u$, that both interaction cases lead to a higher part of the population exhibiting the primacy effect.

4.9

On the contrary, on the right of the figure, for $1.5\ u \leq g < 3\ u$, we note that the interaction case with transmission of congruent features only leads to significantly less people exhibiting primacy effect than in the other cases. This effect takes place for initial attitudes between *U* and 2 *U*, which correspond to the values between the absolute value of the most negative feature *U* and the sum of the absolute values of positive features. Moreover, we note that interactions with transmission of all features do not lead to less primacy effect. In fact, as the aggregated approach shows, an individual having an initial attitude comprised between *U* and 2 *U* and receiving at first a negative feature has still a positive attitude. Due to the emission filter, it does not communicate about its negative retained feature while the others do communicate the positive feature. If it receives a positive feature right after the negative one, it will never be negative. The only possibility to be negative is to receive the two negative features first. The probability of this case decreases because the interactions transmit almost only the positive features.

Impact of the diffusion by the media

4.10

We can think that the frequency of diffusion, which defines how many individuals on average during one iteration are exposed to a feature delivered by the media (parameter *f*), can change the result and suppress the interaction effect. From previous work on the IBM (Deffuant and Huet 2006), we know that for weak frequency of diffusion (0.001 and less), the model tends to yield replicas in which, either all final attitudes are positive, or all are negative. Thus, for a weak frequency of diffusion, the aggregated model cannot be equivalent to the individual based model. However, for higher frequency of diffusion, we can study the persistence of the impact of interactions.

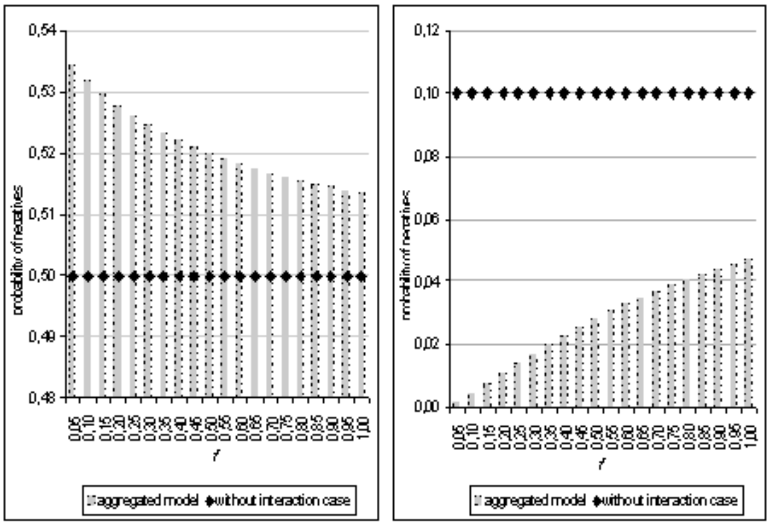

Figure 12. Comparison of the final part of negatives for various values of the frequency parameter *f* in case "without interaction" and case "with interaction" with the aggregated models: on the left, "increase" interaction effect case (for $u \leq g < 1.5\ u$); on the right, "decrease" interaction" effect case at bottom (for $2\ u \leq g < 3\ u$)

4.11

Figures 12 show the sensitivity of the results to variations of *f*. We notice that, the impact of interactions is higher for low frequencies, but even when the frequency is at its maximum value 1, the "increase" or "decrease" primacy effect due to interactions remain. Here the aggregated model gives the possibility to investigate very rapidly the average behaviour of the IBM.

Conclusion, discussion

5.1

We studied an individual based model of "information filtering", which refers to the theory of cognitive dissonance of Festinger and work on rumour from Allport. In this model, for some parameters, the final attitude toward an object depends on the order of reception of the features. This can be interpreted as a variant of primacy effect, because the first received features determine the final attitude. Supposing that a media broadcasts this features in a random order, one can easily predict the final state of a population of isolated individuals. The model is very simplistic, all individuals share the same initial attitude, the same threshold, the same feature values, and therefore its results should be seen as metaphorical. In addition to all these simplifications, we would like to stress a strong hypothesis that could remain unnoticed: all individuals are supposed motivated to process information about the object. Such a situation is very unlikely in real diffusion processes. On the contrary, the majority is often composed of poorly motivated individuals, who tend less to exhibit a primacy effect (Haugtvedt and Wegener 1994).

5.2

When, in addition to the media, individuals can transmit some retained features to their neighbours through an interaction, the outcome is less straightforward to predict. Indeed, for some particular values of the initial attitude *g*, we observe that interactions modify the final part of the population exhibiting primacy effect compared to the case where individuals are isolated.

5.3

To understand better this difference, we build a differential equation model ruling the evolution of probabilities that individuals belong to different groups defined by a set of retained features. We solve it numerically and show that this aggregated model approximates very well the IBM results. Moreover, the analysis of the graph structuring the groups shows how interactions can favour the diffusion of the negative feature.

5.4

This explanation of the increase of primacy effect due to interactions led us to hypothesise that interactions can also decrease the primacy effect. The analysis of the different graphs of groups corresponding to different values of the initial attitude *g* allowed us to identify such a configuration. We checked on both the aggregated model and the IBM that the primacy effect is lower than in the isolated case.

5.5

The double modelling approach provided a significant enrichment for the analysis of the IBM. In particular, it guided us to formulate and verify more easily hypotheses on the IBM behaviour as in Edwards (2004), that we could have missed otherwise. This enrichment comes from the point of view on the dynamics brought by the aggregated model. We consider probability flows between groups instead of individuals. It provides a more compact view of the processes, which eases their understanding.

5.6

Moreover, the aggregated model provides asymptotic results, corresponding to an infinite population. In some cases, such results are useful as a reference.

5.7

However, some specific limitations of the aggregated model should be underlined:

- It is not possible as in Edwards (2003) to globally substitute a single aggregated model to the IBM because the aggregated model graph of groups generally change with the message and the initial attitude.
- For low values of media diffusion frequency, this type of aggregated model is not appropriate.

5.8

In addition, it is possible to derive such aggregated models because the model is simple and has favourable properties. The task can rapidly become impossible when the model becomes more complex.

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