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Analysis of Asymmetric Two-Sided Matching: Agent-Based Simulation with Theorem-Proof Approach

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Abstract

This paper discusses an extended version of the matching problem which includes the mate search problem; this version is a generalization of a traditional optimization problem. The matching problem is extended to a form of the asymmetric two-sided matching problem. An agent-based simulation model is built and simulation results are presented. Todd and Miller (1999) simulated the two-sided matching problem in a symmetric setting. In their model, there are the same number of agents in both parties (groups), each of whom has his/her own mate value. Each agent in a party tries to find his/her mate in the other party, based on his/her candidate's mate value and his/her own aspiration level for his/her partner's mate value. Each agent learns his/her own mate value and adjusts his/her aspiration level through the trial period (adolescence). Todd and Miller (1999) tried several search rules and learning mechanisms that are symmetric for both parties. In the present paper, Todd and Miller's (1999) model is extended to an asymmetric setting where the two parties have different numbers of agents, and the search rule and the learning mechanism for the two parties differ. Through the simulation, the search rules and the learning mechanisms which were identified to be appropriate in a symmetric setting are revealed to be inappropriate in the asymmetric setting and the reason why this is so is discussed. Furthermore, some general facts are derived using a mathematical theorem-proof approach. Some of these facts are used to direct a revision of the model, and a revised simulation model is presented. An implication is obtained for practical situations in asymmetric matching setting. For example, in the job hunting case, if job applicants want to finish their job hunting successfully, they should be modest at the beginning of the hunt.

Keywords:

Social Simulation, Agent-Based Models (ABM), Theorem-Proof Approach, Mate-Search Problem, Two-Sided Matching, Job Matching



Introduction

- 1.1 There is a well-known optimal stopping problem called the "Secretary Problem". A set of applicants for a secretary position is given to an employer who is searching for a secretary. Each applicant has a value that denotes how good the applicant is for the employer. One can think of the value as an index that indicates the capability of carrying out a secretary's job. Therefore, the problem is how to find an employee with as high a value as possible.
- 1.2 The employer cannot know the value of all applicants in advance, so he/she has to examine each applicant's value by interviewing them one by one. In addition, the employer has to decide whether to hire each applicant at the time of the interview. The employer is not allowed to change his/her mind afterward. Therefore, once the employer decides to refuse the applicant, he/she can never hire that person.
- 1.3 There is only one job opportunity. Hence, once the employer decides to hire the applicant, he/she has to stop interviewing, even though there are applicants left who have not been interviewed. So, the Secretary Problem is the challenge of finding the rule for deciding whether to accept or refuse each interviewed applicant, and when to stop the job interviews in order to employ the applicant having the highest value with as high probability as possible.
- 1.4 In the Secretary Problem, the optimal solution is called the "37% rule" (Ferguson 1989). The rule is as follows. First, conduct interviews one by one to examine the value of a 37%(= $1/e$) sample of the entire set of applicants. The applicants who are picked

as the sample are interviewed, but not employed. Call this stage the “sampling period”. Secondly, enter the actual interview period when the interviews of the sample applicants are finished, remembering the highest value observed in the sampling period. In the actual interview period, once an applicant with a higher value than the highest value in the sample is encountered, decide to accept that applicant as the secretary and stop the interviews. (This part of the rule ensures that the probability of hiring the applicant with the best value is as high as possible.) If an applicant with a value higher than the best one in the sample is not encountered, hire the last interviewee, no matter how small his/her value is. (In this case, the employer has failed to hire the secretary with the highest value.)

- 1.5 There is a problem called the “Dowry Problem”, or the “Mate Search Problem”, which is an interpretation of the Secretary Problem in another context. This problem is to determine how one finds a mate having the highest value. This interpretation is useful because it inspires us to extend the problem to a two-sided search problem. Generally speaking, a mate search is not one-sided, as in the Secretary Problem case, where only the employer is trying to optimize the value of another person (the employee); rather it is two-sided in that a matching that is desirable to both sides is required. Todd and Miller extended the Secretary Problem to a two-sided matching problem and examined several search rules by computer simulation (Todd 1997; Todd and Miller 1999). They proposed several rules under assumptions that are appropriate for the usual mate search situation and compared them to find a desirable search rule in accordance with certain criteria, which will be described below in detail.
- 1.6 Mate search activities involve building a one-to-one matching between two parties, say, men and women. Therefore, the condition for searching can be set symmetrically for both parties. However, there are matching situations where this symmetry is not appropriate. For example, job hunting is a problem in which each agent in one party (firms) tries to find many member of the other party (job applicants). A special problem occurs in Japan, where almost all colleges end their academic year in March, and most graduates start their jobs in April. Most college students in Japan start their job hunting about one year ahead of their graduation. Additionally, it is common for firms in Japan to employ new graduates as almost all of their new employees. This custom is called “simultaneous recruiting of new graduates”. Therefore, in Japan this matching takes place between a large number of firms and students simultaneously. This custom causes many social problems in Japan (Economist 2012). In recent years, graduating students in Japan have ended their job hunting without receiving any job offers from firms, despite many firms not fulfilling their recruitment targets. Once they fail in job hunting prior to graduation, it becomes very difficult for them to obtain job offers because most firms in Japan employ only new graduates. Examining why the mismatch between students and firms occurs can help solve this problem.
- 1.7 We say a two-sided matching situation is asymmetric when properties of the two parties (the number of agents and the number of acceptable mates for each agent) and the decision rules of agents (how they decide their mates and when to stop their search) in two parties differ. In this paper, we try to extend Todd and Miller’s (1999) simulation model to an asymmetric situation and examine one of their search rules under asymmetric conditions.



Two-sided matching in the literature

- 2.1 Two-sided matching has been well analyzed using game theory. In those discussions, the existence of stable matching solutions was tackled and ways of finding them were sought. Many theories have been developed within the game theoretical framework (Roth and Sotomayor 1992). Stable matching theory, pioneered, by Gale and Shapley (Gale and Shapley 1962), was used to study the formation and dissolution of marriages by Mumcu and Saglam (2008) under the assumption of transferable utilities and by Saglam (2011) under the assumption of nontransferable utilities. These assume two parties, such as men and women, colleges and students, or firms and workers, and also assume that the preferences of each agent for agents in the opposite party are given. A stable matching solution refers to a matching at which no pair of agents from the two parties who are not matched prefer each other. It is known that stable matchings exist in many situations and an algorithm called the “deferred acceptance” procedure can give one of the stable matchings (Roth 1984a; Gale and Shapley 1962). Using this procedure, Roth analyzed operations in the labor market, for example, the situation in which intern medical students are allocated to internships in hospitals in America (Niederle et. al 2008; Roth 1984b). In these models, they assume that agents are able to hold the best offer they have received, without accepting it outright (Roth and Sotomayor 1992), and they focus on finding a procedure (an algorithm) which gives stable matchings. Additionally, in most cases they assume the existence of a centralized clearinghouse which collects agents’ preferences and runs the algorithm, and try to answer the question, “how can we design the procedure which the clearinghouse should follow?”.
- 2.2 However, in the job hunting situation in Japan, these assumptions do not hold. Job allocation is carried out on an individual basis; therefore, we cannot assume the existence of centralized clearinghouses. Furthermore, potential workers most of whom are university students have to decide whether to accept offers from firms within a short time; therefore, they cannot hold the best offer to defer their answer to firms. The assumption that each agent can present his/her preferences to the agents in the opposite party before the allocation procedure starts does not apply, either. The number of potential workers is very large and each of them is unknown to firms before the job hunting starts. The number of potential firms who would give offers to workers is also large for each worker, and university students who apply to job vacancies do not know the precise value of firms until they apply. Therefore, we cannot rely on the game theoretical framework which requires agents’ preferences in advance.
- 2.3 As was mentioned in the previous section, Todd and Miller (1999) carried out a simulation based on agents’ autonomous behavior in the marriage mate search. Their target was a symmetric two-sided matching situation, in which both the numbers of

agents in each party and also the mate search strategies of the two parties are the same. However, the job hunting situation is asymmetric in the sense that the numbers of agents differ because the number of job seeking students exceeds the number of firms, and the information held by individuals also differs between parties because firms know their own values better than job applicants know their own values. Therefore, the search strategies in the two parties should be different. As far as the author is aware, no agent-based simulation of this type of asymmetric two-sided matching situation has been reported. Hence, we will review Todd and Miller's (1999) symmetric model in brief and then extend it to the asymmetric situation.



Symmetric two-sided matching

- 3.1 In this section, we formalize Todd and Miller's (1999) model of the symmetric two-sided matching problem. We will extend this model to an asymmetric setup in the next section.

Model

- 3.2 Let M and W be the sets of men and women, respectively. We assume that the number of people in each set is identical, say n . Hence, $|M| = |W| = n$. We designate M and W as

$$M = \{1, 2, \dots, n\},$$

$$W = \{n+1, n+2, \dots, 2n\},$$

and call each element of these two sets an agent. Each agent $k \in \{1, 2, \dots, 2n\}$ has two attributes. One is the mate value v_k , which is the value of the agent as a mating partner. The other is the aspiration level a_k for a mating partner. Agents' own mate values do not vary throughout mate search. On the other hand, a_k changes, based on rules that will be described below, while agent i searches for his/her mate. Hereinafter, we assume that $v_k \in [0, 100]$. These values are randomly assigned to agents in the simulations.

- 3.3 Agents search for mates in two sequential periods.

1. Learning (adolescence) period

In this period, each agent in M and W exchanges information with some of the agents of the other party. In other words, the agents date each other. In this step, agents update their aspiration levels following a learning rule which is given in advance. They do not actually mate in this period. This period corresponds to the sampling period in the Secretary Problem.

2. Mating period

In this period, the actual mating takes place. If an agent finds a partner with a value greater than the aspiration level that he/she acquired in the learning period, then the agent proposes to the partner. If both of a pair propose to each other, they succeed in mating and stop their search. Agents who cannot mate with any candidate end up failing to find a mate. When all agents either succeed or fail, this period finishes.

- 3.4 Let c (%) denote the sampling ratio, the ratio of the number of people whom each agent can date within the learning period to the number of potential mating candidates n . Therefore, all agents date with $n^*c/100$ people in their adolescence period. They cannot mate with candidates whom they once dated. This is the same assumption as in the Secretary Problem.
- 3.5 Todd examined the following rules for learning the aspiration level in the learning period. We describe how the aspiration level changes through dating based on each learning rule when a man $k \in M$ and a woman $j \in W$ date each other. We denote the aspiration level of agent k ($= i, j$) before dating by a_k , and that after dating by a'_k . The learning rules are symmetric between men and women. Therefore, we show the learning rules from the men's side only.

Take the Next Best (TNB) rule:

This is a generalization of the 37% rule in the Secretary Problem. This does not fix the sampling ratio to 37%. We can change it by altering c .

$$a'_i = \begin{cases} v_j & (\text{if } v_j > a_i) \\ a_i & (\text{otherwise}) \end{cases}$$

Hence, when an agent i dates an agent j with a higher mate value v_j than his current aspiration level a_i , he changes his aspiration level from the current value a_i to the mate value v_j of his date j . This means that he remembers the maximum mate value from among those he has previously dated. We assume that the initial value of a_i is 0.

Mate Value-5 rule:

This sets the aspiration level to

$$a_i = v_i - 5,$$

and does not change it throughout the learning period. An agent with this rule is modest and does not have high aspirations. This rule assumes that he knows his own mate value beforehand. However, in general it is difficult for people to know accurately their mate value: few people know their own appeal to the opposite gender. The following rule assumes that agents cannot know accurately their mate value, and that they learn and approximate this value through dating and exchanging information with agents of the opposite gender.

Adjust Relative/2 rule:

This changes the aspiration level depending on whether the date proposes. "To propose" in the learning period is just to express that "You are my favorite"; here, agents do not actually mate. In addition, this rule adjusts the aspiration level depending on whether the proposing date's mate value is higher than the agent's own mate value, and vice versa when he is not proposed to. Actually, we are assuming that they cannot know accurately their own mate value, and therefore they use their aspiration level based on the assumption that their aspiration levels approximate their mate values well. The adjustment width varies depending on the difference between the date's value and the agent's own value (precisely speaking, their aspiration level), as follows:

$$a'_i = \begin{cases} a_i + \Delta & (v_i \geq a_j \text{ and } v_j \geq a_i) \\ a_i - \Delta & (v_i < a_j \text{ and } v_j < a_i) \\ a_i & (\text{otherwise}) \end{cases},$$

where $\Delta = |v_j - a_i|/2$.

When proposed to ($v_i \geq a_j$), and the date's value is greater than or equal to the approximation of his own mate value ($v_j \geq a_i$), agent i adjusts his aspiration level upward. Δ is the adjustment width used in raising or lowering the aspiration level. Therefore, the adjustment is carried out to narrow the gap between his own aspiration level and the date's value. The initial value of a_i when the learning period starts is set to 50, the middle of the mate value range.

- 3.6 Todd and Miller (1999) presented several variations of the above adjustment rule and concluded that the above rule is the most sophisticated and the best one among these variations. We accept their conclusion and adopt this rule to model agents' learning of their own values in the present paper.

Evaluation criteria of search rules

- 3.7 Todd and Miller (1999) claim that the desirable search rule (or learning rule) should meet the following criteria (Todd and Miller 1999).
1. It generates as many matings as possible. In other words, the greater the total number of pairs generated by a successful mating, the better the rule.
 2. The mate values of agents who end up successfully mating are not biased as too high or too low. In other words, the average mate value of those who successfully found their mates should be near the middle of the mate value range, i.e., 50.
 3. The difference in mate values between the man and woman in the pair of a successful mating is not too large. In other words, the lower the average of the difference of mate values between the two agents in pairs generated by a successful mating, the better the rule.

Under these criteria, Todd and Miller (1999) conducted and evaluated rules above by computer simulation and concluded that the Adjust Relative/2 rule is the most desirable.

- 3.8 The author replicated Todd and Miller's (1999) simulation model written in C language and NetLogo, and confirmed their conclusions as follows. Readers can download the NetLogo version replicated by the author from the OpenABM model archive (<http://www.openabm.org/model/3575/version/2/>). The TNB rule generates markedly fewer successful matches. And as was described above, the Mate Value-5 rule is based on the unrealistic assumption that each agent knows accurately his/her own mate value. The result that the Adjust Relative/2 rule is the best comes from its satisfying the three criteria described above. In particular, with the Adjust Relative/2 rule the average value of successful agents approaches 50 and the average of the value difference of successful matchings decreases as the learning period lengthens. Hence, we can conclude that the Adjust Relative/2 rule is the most sophisticated rule in the situation that agents do not know accurately their own values beforehand and have to learn them in their learning period. Confirming this result, we will extend Todd and Miller's (1999) model to asymmetric situations.



Asymmetric two-sided matching

- 4.1 We extend Todd and Miller's (1999) model to apply it to the situation of asymmetric two-sided matching and then conduct computer simulations.

Model

- 4.2 Let A and F be the set of applicants and the set of firms, respectively. Let the number of elements in A and F be m and n , respectively. We assume that $m > n$. As in the previous section, we call the elements of A and F agents. And also, each agent $i \in \{1, 2, \dots, m+n\}$ has its own value v_i and aspiration level a_i w.r.t. the value of matching partners. The firms' values indicate the degree of firms' desirability from an applicant's perspective. On the other hand, applicants' values indicate the applicants' capabilities of carrying out jobs in firms. We sometimes refer to an applicant's values as a capability value. Each firm agent has a recruitment number (number of applicants to be recruited) as an attribute. For simplicity, we assume that all firm agents have the same recruitment number, denoted by l . Furthermore, we assume that

$$m = ln,$$

that is to say, the total job vacancies being recruited for and the number of applicants seeking jobs are the same. We should note that this assumption is independent of the assumption of asymmetry. We are requiring this assumption to ensure that all applicants can find jobs and all job vacancies can be filled in principle. The actual job market in Japan can support this assumption because the recent ratio of job vacancies to job applicants has been around 1.2 (Recruit Works Institute 2013).

- 4.3 Firms interview each applicant applying to be recruited by them, decide whether to accept or to refuse him/her, and tell the applicant their decision. Once a firm gives a decision of refusal, it is not allowed to recruit the refused applicant later, even if the firm still has an unfilled recruitment position. Similarly, an applicant cannot reapply to a firm that has already refused him/her.
- 4.4 In the symmetric two-sided matching case, agents in both parties learn their aspiration level. In the asymmetric situation, we assume the following.

1. Each firm $i \in F$ has already learned their value v_i .
2. Firms set their aspiration level a_i lower than their value v_i .

These two assumptions can be supported by the following reasons. First, the recruitment performed by firms is not a one-shot activity, but is repeatedly carried out. Therefore, we can assume that they have learned their values through successive recruitment activities. Second, we are assuming that the total number of job vacancies is equal to the number of applicants. If the agents' values for both parties are randomly set, it is unavoidable that some applicants end up with being unsuccessful. If firms try to fill as many recruitment positions as possible, under this condition, they will have to set their aspiration levels lower than their own values to some extent. Therefore, we assume that all firms adopt the Mate Value-5 rule described above. As was described in the previous section, this rule is based on the two assumptions listed above.

- 4.5 On the other hand, applicants do not necessarily know accurately their values. Therefore, they have to learn their values and adjust their aspiration levels in their learning period. As was confirmed by Todd and Miller (1999.) in the symmetric agents model, Adjust Relative/2 is the most sophisticated rule Todd and Miller (1999). Hence, we assume that all applicants adopt the Adjust Relative/2 rule for learning their aspiration level.

Simulation

- 4.6 Based on the assumptions described above, we conduct computer simulations with the Mate Value-5 rule for firms and the Adjust Relative/2 rule for applicants under the following parameter settings. The author built two versions of the extended model written in C language and NetLogo. Readers can download the NetLogo version built by the author from the OpenABM model archive (<http://www.openabm.org/model/3577/version/2/>).

Number of firms: $m = 100$

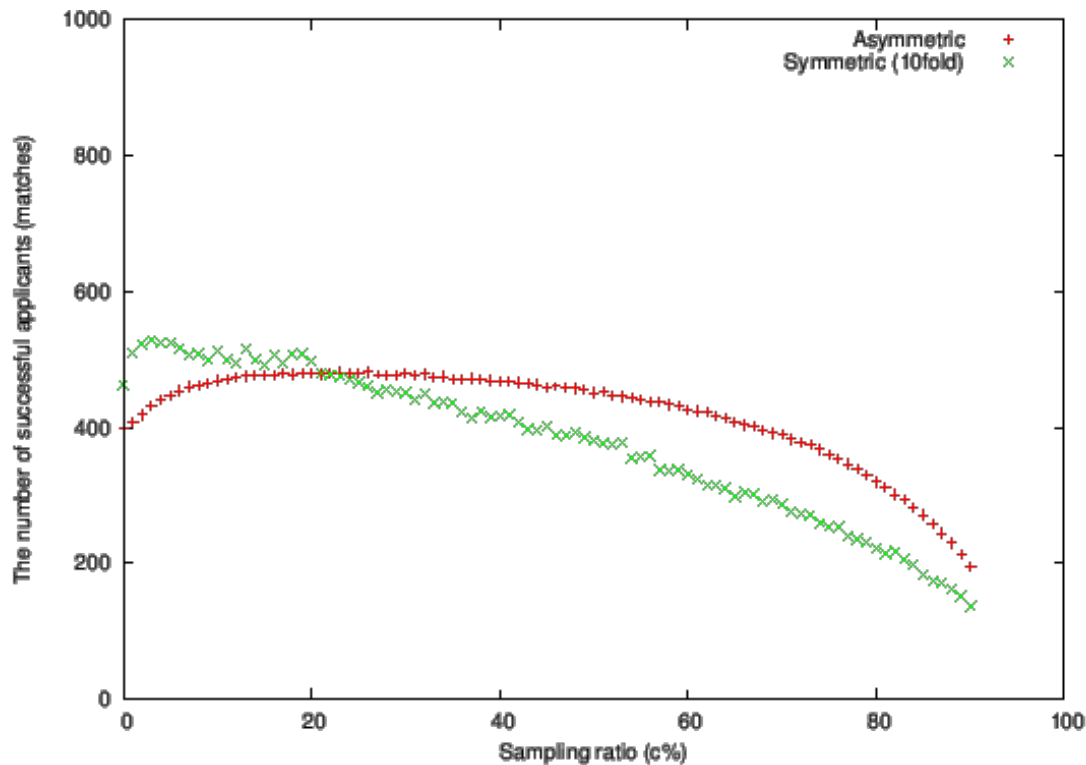
Number of applicants: $n = 1000$

Recruitment number of each firm: $l = 10$

- 4.7 Each agent's value is randomly set to an integer taken uniformly from the interval $[0, 100]$. We performed the simulations for a sampling ratio $c\%$ taken as the integers from 0 to 90. Figure 1 through 3 respectively show plots of the following quantities at the end of the matching process:

1. Number of successful applicants (Figure 1)
2. Average value of successful applicants (Figure 2)
3. Average difference in value between an applicant and a firm in a successful matching (Figure 3)

The parameter setting $m = 100$ implies that the length of the learning period of applicants (number of sample firms) is $c (= m \cdot c/100)$. Hence, the horizontal axis of each plot is also the length of the learning period. Ten simulations were executed with different random seeds for each sampling ratio, and the mean value of these 10 executions are plotted in the three figures. In the figures, both the asymmetric and the symmetric (Adjust Relative/2) models are shown for comparison. Note that the values for symmetric matching in Figure 1 are multiplied by 10 because the number of possible matches is 100 in the symmetric case, where n (number of agents in each party) is 100.



(*) Number of successful matches in the symmetric model is multiplied by 10.

Figure 1: Number of successful applicants (matches*)

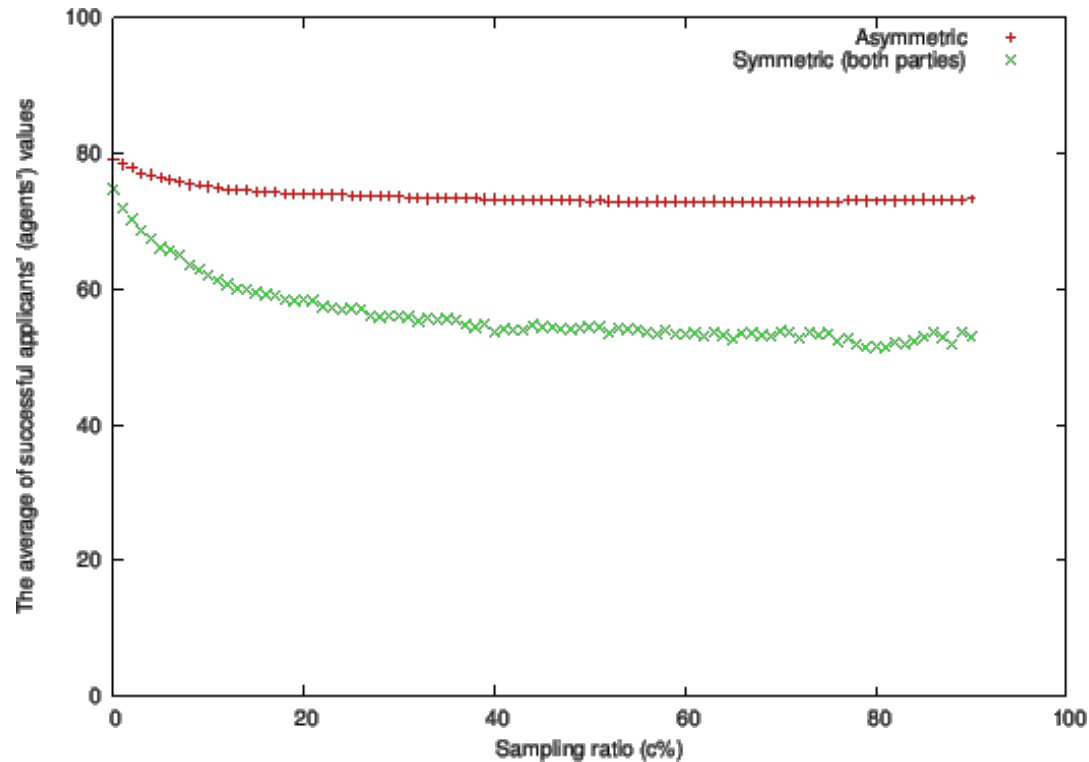


Figure 2: Average value of successful applicants (agents)

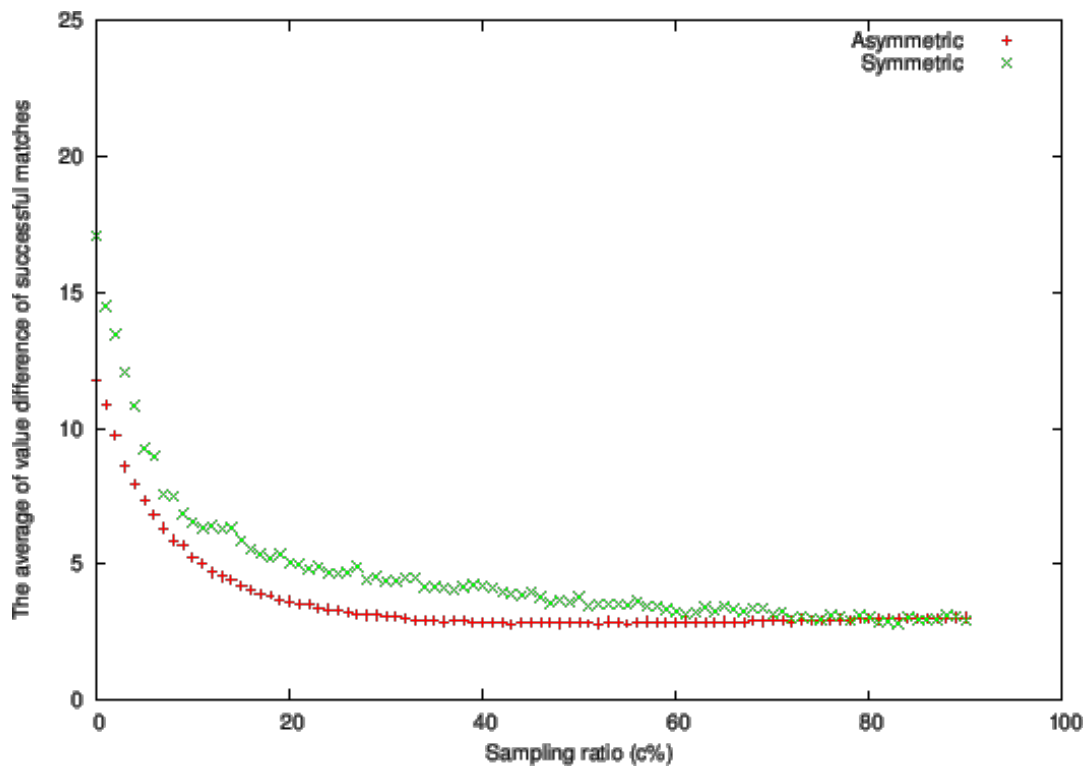


Figure 3: Average difference in values for in successful matchings

- 4.8 Comparing with Adjust Relative/2 in Todd and Miller's (1999) symmetric situation, we do not observe a significant change in the number of successful agents (Figure 1) or the average difference in value (Figure 3). However, the average agent value (Figure 2) does exhibit a significant difference from Todd and Miller's symmetric situation. In Todd and Miller's model, the average value of successful agents with the Adjust Relative/2 rule converges to about 50 as c increases, giving agents more learning opportunity. This means that both the agents with higher values and the ones with lower values have similar levels of success at matching. However, in the asymmetric situation with the Adjust Relative/2 rule, the average only converges to about 70. Therefore, in the asymmetric case, more applicants with low values end up failing in their job hunting than those with high values.
- 4.9 Having observed these results, we take a theorem-proof approach in order to find the reason why they occur.



Theorem-proof approach and revised simulation model

- 5.1 We next try to find the reason why the matching with the Adjust Relative/2 rule did not work well in asymmetric situations. The agent-based simulation approach usually traces back through the simulation process and discusses the reasons for the observed results, what is called "debriefing". In the present paper, we take another approach, the theorem-proof approach. Some facts that hold in our model will be revealed in this section.
- 5.2 In the simulation, we use the Mate Value-5 rule for firms. However, the numeral "5", which denotes the degree of modesty used, does not affect the following discussion. Therefore, we generalize the rule as Mate Value- α for any $\alpha > 0$.
- 5.3 We also assume the following in this section.
1. Firms adopt the Mate Value- α ($\alpha > 0$) rule. The parameter α is common to all firms. Hence, $a_j < v_j$ always holds for any firm $j \in F$, and $v_j - a_j = \alpha$.
 2. Applicants adopt the Adjust Relative/2 rule.

Propositions

- 5.4 Some terms should be defined in order to characterize the behavior of applicants that affect their performance in job hunting.

Definition Idealistic/realistic agent

Let $i \in A$ be any applicant. If he/she aspires to firms with higher values than his/her capability, i.e., $a_i > v_i$, we call him/her an idealist. Otherwise ($a_i \leq v_i$), we call him/her a realist.



- 5.5 In the following propositions, the difference in job hunting behavior between idealistic agents and realistic ones will be shown. In order to make this paper more readable, all of the proofs of the propositions and theorems are placed in the appendix.

Proposition 1 *If an applicant is an idealist, then he/she remains idealistic after an adjustment.*

5.6 This proposition says that idealistic applicants do not become realists through the learning process with Mate Value- α firms.

5.7 The next two propositions refer to how the degree of idealism changes because of an information exchange with a firm.

Proposition 2 *For any idealistic agent $i \in A$, if $|a_i - v_i| > \alpha$, then $|a'_i - v_i| > \alpha$.*

Proposition 3 *For any idealistic agent $i \in A$, if $|a_i - v_i| \leq \alpha$, then $|a'_i - v_i| \leq \alpha$.*

5.8 Therefore, an idealist with a large degree of idealism (i.e., $|a_i - v_i| > \alpha$) remains the same after each interaction, and so does one with a small degree (i.e., $|a'_i - v_i| \leq \alpha$).

5.9 The next proposition refers to a realist's behavior.

Proposition 4 *If any applicant $i \in A$ is a realist, then he/she can become an idealist after an adjustment. A realist agent i becomes an idealist after an adjustment involving firm $j \in F$ iff $v_j < (a_i + v_i)/2$ holds. Whenever this condition is satisfied, $|a'_i - v_i| < \alpha/2$ holds.*

5.10 The above proposition says that a realist can become an idealist when a condition on his/her value, his/her aspiration level, and the firm's value is satisfied. However, the degree of idealism after learning is small relative to α .

5.11 Combining the above four propositions, we can state the following theorem.

Theorem *For any applicant agent $i \in A$, the following hold.*

1) *If $v_j < a_i$ and $|a_i - v_i| > \alpha$, then $v_j < a'_i$ and $|a'_i - v_i| > \alpha$.*

2) *If $a_i \leq v_i$ and $v_j < a'_i$, then $|a'_i - v_i| < \alpha/2$.*

5.12 We can restate the theorem in our target situation, job hunting, as follows.

Corollary 1) *Any idealistic applicant remains idealistic with a large degree of idealism throughout the learning period if he/she is an idealist with a large degree of idealism at the beginning of the learning period.*

2) *Any realistic applicant can become idealistic in the learning period. In this case, however, he/she remains idealistic with a small degree of idealism for the rest of his/her learning period.*

5.13 It is obvious that it is harder for a idealistic applicant to find a job than for a realistic one. We set the initial values of the aspiration level for applicants as the middle value of the range, 50. This implies that applicants with capability values under 50 remain idealistic throughout their job hunting, and it is thus more likely that they will fail in their job hunting. On the other hand, applicants with capability values over 50 start their learning period as realists. Some of them remain realists throughout the job hunting process, whereas other realists become idealists. These converted applicants remain idealists with a small degree of idealism throughout their job hunting. These facts generate the observed bias in the average capability values of successful applicants.

5.14 These considerations suggest that we can correct the observed bias by using a new search rule which sets the initial aspiration level of applicants lower than their capability values. We name this rule "Adjust Relative/2 with Low Initial Aspiration". However, as was mentioned above, applicants do not know their own capability values when they start their job hunting. Therefore, we set the initial aspiration as zero.

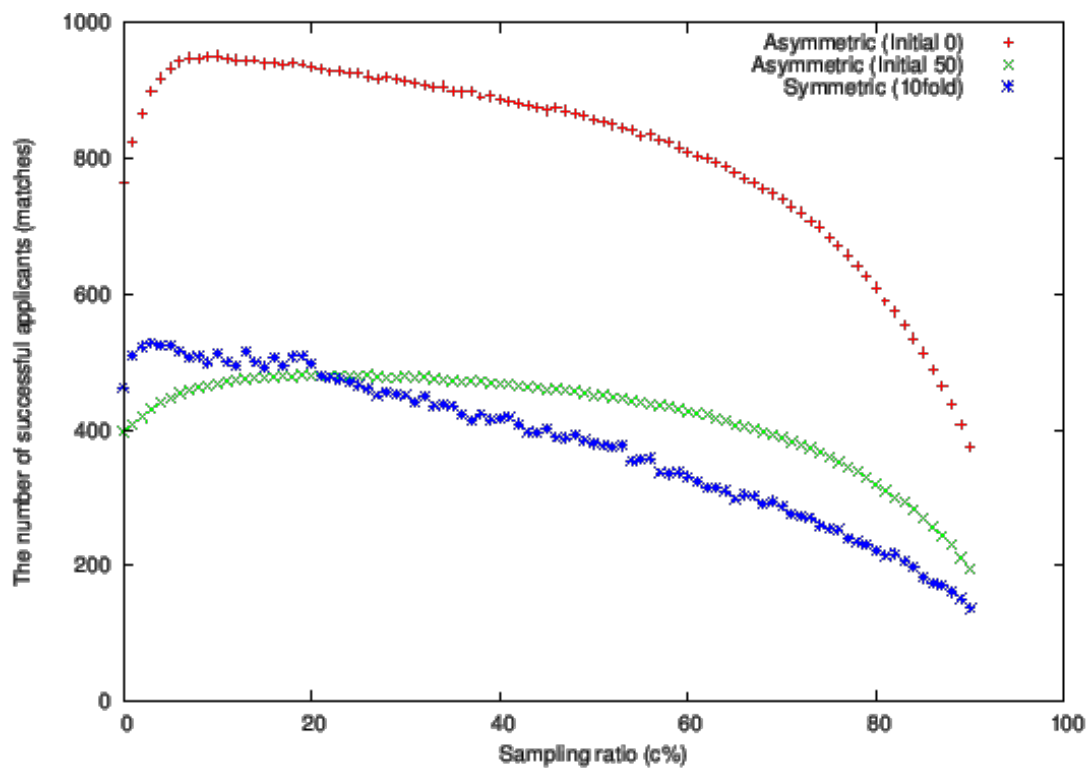
Adjust Relative/2 with Initial 0 rule:

This uses the same rule for adjustment as the one in the Adjust Relative/2 rule except for the initial value zero of aspiration a_i for all $i \in A$.

5.15 We refer to what was formerly called the "Adjust Relative/2" rule as the "Adjust Relative/2 with Initial 50" rule to distinguish it from the new rule. In the next section, we revise the simulation settings and verify the conjecture that the modified rule can correct the observed bias.

Revised simulations

5.16 Simulations were carried out with the same settings as in Section 4 except that the initial aspiration levels of applicants are all set to zero. Therefore, all applicants start their learning as realists. Figure 4 through 6 show the results for the asymmetric model with initial zero and that with initial 50 aspiration levels. They also show the results for the Todd and Miller's symmetric model for comparison. The plotted values for the symmetric matching in Figure 4 are multiplied by 10, as was done in the case of Figure 1.



(*) Number of successful matches in the symmetric model is multiplied by 10.

Figure 4: Number of successful applicants (matches*)

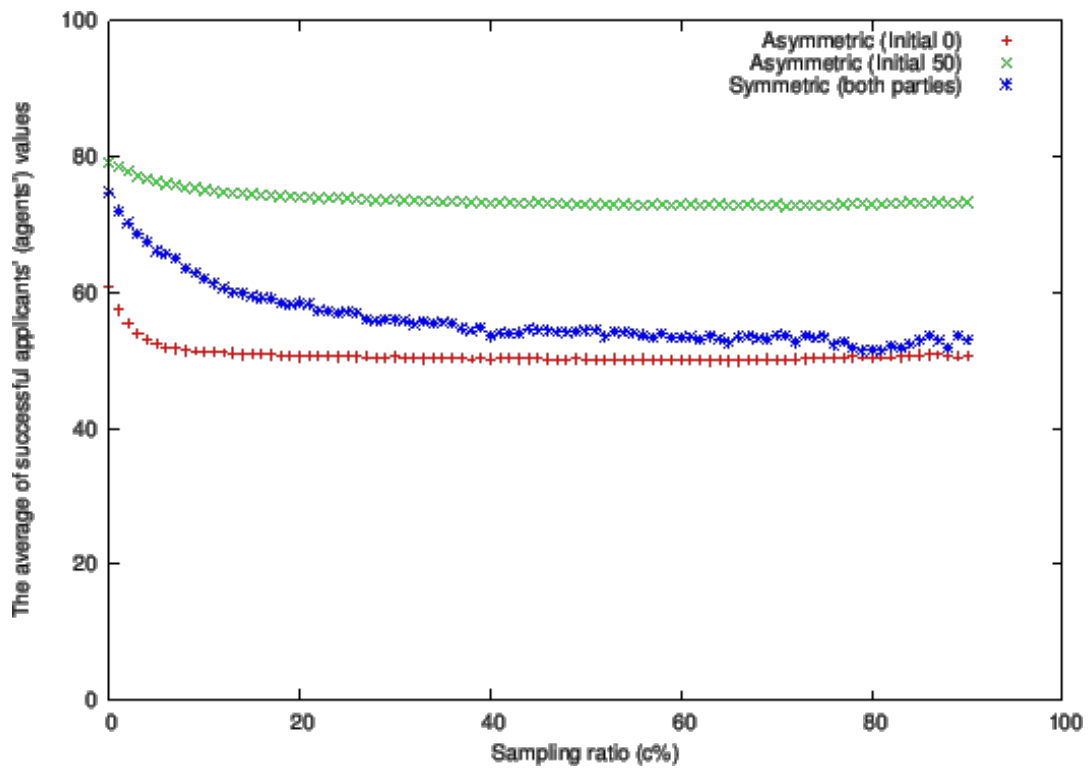


Figure 5: Average value of successful applicants (agents)

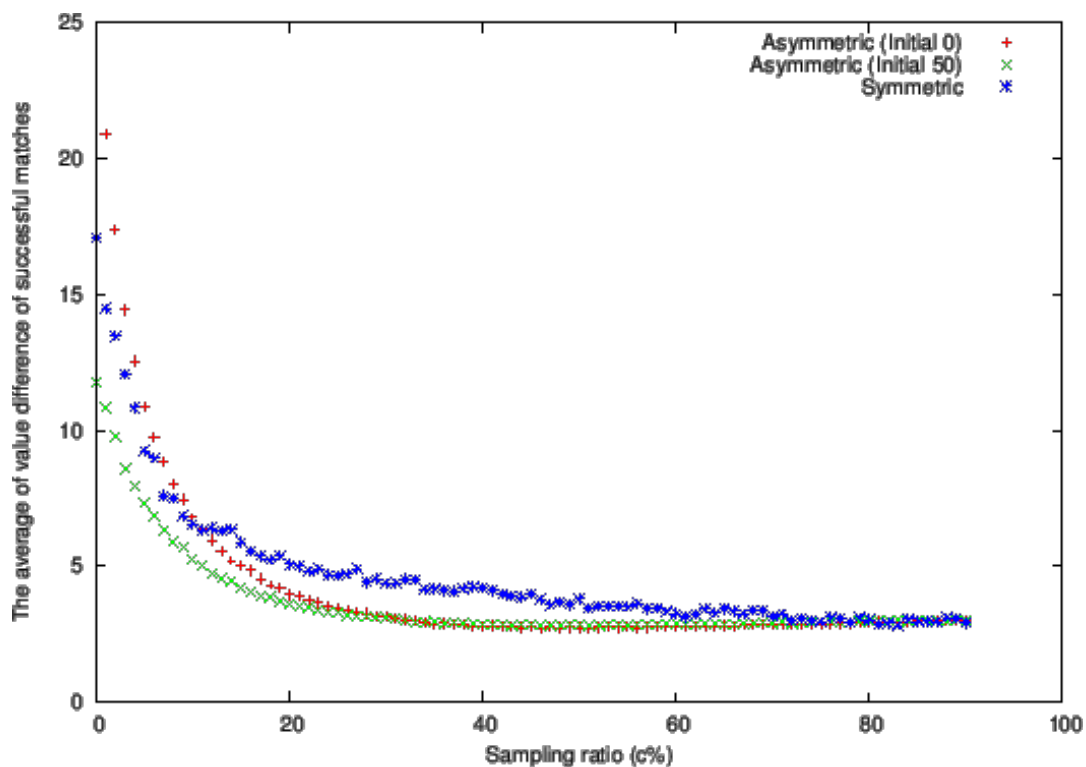


Figure 6: Average difference in the values in successful matchings

- 5.17 Figure 4 reveals that the number of successful applicants increased dramatically relative to the Initial 50 case. Figure 5 shows that the average value of successful applicants converges to about 50. Hence, both high-valued and low-valued applicants succeeded in their job hunting. There is little difference between the three setting for the average of value difference of successful matching (Figure 6). This implies that the difference between firms' values and applicants' values remained small.



Conclusion

- 6.1 We tried a hybrid method of combining agent-based simulations and a mathematical theorem-proof approach for the asymmetric two-sided matching problem. Todd and Miller's (1999) symmetric model was extended to an asymmetric situation, creating a framework that can be interpreted as a model of the job hunting situation.
- 6.2 In the simulations, firms are assumed to know their own values precisely and to set their aspiration levels lower than their values (Mate Value-5 rule) in order to fulfill their recruitment targets. Whether the Mate Value-5 rule (or the Mate Value- α rule in general) is the optimal adjustment rule for the firms, given the search rule of the applicants (Adjust Relative/2 rule), in the described matching environment is the subject of future research. Under the assumptions we made, applicants with lower capability values tend to fail relatively more in their job hunting. We showed the reason for this simulation result by using a theorem-proof approach. Applicants with lower capability values start their job hunting as idealists. Since idealists remain idealists throughout the job hunting process, more such applicants fail to find their jobs than higher capability applicants. On the other hand, some realists become idealists, but with a consistently low degree of idealism. This generates the biased result observed in our simulations.
- 6.3 These considerations have implications for college students in Japan who are preparing for job hunting. If they want to finish their job hunting successfully, they should follow our proposed new rule, the relative adjustment with low initial aspiration. That is, they should be modest at the beginning of their job hunting.



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Appendix

Proof of Proposition 1

Proposition 1 *If an applicant is an idealist, then he/she remains idealistic after an adjustment.*

Proof:

Let $i \in A$ and $j \in F$ be any applicant and any firm, respectively. Assume that $a_i > v_i$. Let a'_i denote the aspiration level of i after exchanging information with j . It suffices to show that $a'_i > v_i$. When adjustment of a_i does not take place, we have $a'_i = a_i$ and there is nothing to prove. Adjustment of the aspiration level a_i of the applicant occurs only in the following two cases.

1. Upward adjustment ($a'_i \geq a_i$) when $v_i \geq a_j$ and $v_j \geq a_i$
2. Downward adjustment ($a'_i < a_i$) when $v_i < a_j$ and $v_j < a_i$

In case 1, we have $a'_i > v_i$. In case 2, the Adjust Relative/2 rule ensures that $v_j < a'_i < a_i$, and we also have $v_i < a_j$. With the assumption that j adopts the Mate Value- α rule, we have $a_j < v_j$. Therefore, we can conclude that $v_i < a'_i$.

Q.E.D.

Proof of Proposition 2

Proposition 2 *For any idealistic agent $i \in A$, if $|a_i - v_i| > \alpha$, then $|a'_i - v_i| > \alpha$.*

Proof:

In the proof of Proposition 1, case 1 guarantees that

$$|a'_i - v_i| > |a_i - v_i| > \alpha.$$

For case 2, we have

$$v_i < a_j < v_j < a'_i.$$

Together with $|v_j - a_j| = \alpha$ from the firm behavior, it follows that $|a'_i - v_i| > \alpha$.

Q.E.D.

Proof of Proposition 3

Proposition 3 *For any idealistic agent $i \in A$, if $|a_i - v_i| \leq \alpha$, then $|a'_i - v_i| \leq \alpha$.*

Proof:

Consider any idealistic agent $i \in A$. Then $a_i > v_i$. If a downward adjustment takes place, we have $a'_i < a_i$. Since $v_i < a'_i$ holds from Proposition 1, it follows that

$$|a'_i - v_i| < |a_i - v_i| \leq \alpha.$$

Hence, it suffices to prove that $|a'_i - v_i| \leq \alpha$ when an upward adjustment takes place. This adjustment takes place when $v_i \geq a_j$ and $v_j \geq a_i$ holds. Since we have $a_i \leq a'_i \leq v_j$ from the Adjust Relative/2 rule, it follows that

$$|a'_i - v_i| \leq |a'_i - a_j| \leq |v_j - a_j| = \alpha.$$

This completes the proof.

Q.E.D.

Proof of Proposition 4

Proposition 4 *If any applicant $i \in A$ is a realist, then he/she can become an idealist after an adjustment. A realist agent i becomes an idealist after an adjustment involving firm $j \in F$ iff $v_i < (a_i + v_j)/2$ holds. Whenever this condition is satisfied, $|a'_i - v_i| < \alpha/2$ holds.*

Proof:

Assume that $a_i \leq v_i$. An adjustment occurs only in the following two cases.

1. Upward adjustment ($a'_i \geq a_i$) when $v_i \geq a_j$ and $v_j \geq a_i$
2. Downward adjustment ($a'_i < a_i$) when $v_i < a_j$ and $v_j < a_i$

However, in the current situation, case 2 cannot take place, because we have $a_j < v_j$, and if the conditions of the case 2 held, we would have

$$v_i < a_j < v_j < a_i,$$

which contradicts that $a_i \leq v_i$.

Assume that an upward adjustment occurs under the condition stated as case 1. Then we have

$$a'_i = a_i + \frac{v_j - a_i}{2} = \frac{a_i + v_j}{2}.$$

Hence, we have a necessary and sufficient condition for $a'_i > v_i$,

$$v_i < \frac{a_i + v_j}{2}.$$

Finally, we assume that $v_i < \frac{a_i + v_j}{2}$. We know that $v_i < a'_i$. Hence, we only have to show that $a'_i - v_i < \alpha/2$ to prove that $|a'_i - v_i|$

$< \alpha/2$. We have two cases to prove.

(Case I) Assume $a_i < a_j$.

$$\begin{aligned} a'_i - v_i &\leq a'_i - a_j \quad (\text{because } a_j \leq v_i) \\ &= \frac{a_i + v_j}{2} - a_j \\ &= \frac{a_i - a_j}{2} + \frac{v_j - a_j}{2} \\ &= \frac{v_j - a_j}{2} - \frac{a_j - a_i}{2} \\ &< \frac{v_j - a_j}{2} \quad (\text{because } a_i < a_j) \\ &= \frac{\alpha}{2} \end{aligned}$$

(Case II) Assume $a_j \leq a_i$.

$$\begin{aligned} a'_i - v_i &\leq a'_i - a_i \quad (\text{because } a_i \leq v_i) \\ &= \frac{a_i + v_j}{2} - a_i \\ &= \frac{v_j - a_i}{2} \\ &\leq \frac{v_j - a_j}{2} \quad (\text{because } a_j \leq a_i) \\ &= \frac{\alpha}{2} \end{aligned}$$

Therefore, we have that $a'_i - v_i \leq \frac{\alpha}{2}$. But the relation does not hold with equality because $a_j < v_i$ implies that $v_i = a_i = a_j$ does not occur and two equalities in the above derivation cannot hold simultaneously.

Q.E.D.

Theorem For any applicant agent $i \in A$, the following hold.

- 1) If $v_i < a_i$ and $|a_i - v_i| > \alpha$, then $v_i < a'_i$ and $|a'_i - v_i| > \alpha$.
- 2) If $a_i \leq v_i$ and $v_i < a'_i$, then $|a'_i - v_i| < \alpha/2$.

Proof:

- 1) Obvious from Proposition 1 and Proposition 2.
- 2) Obvious from Proposition 3 and Proposition 4.

Q.E.D.



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